62[K, P, X].—L. RADANOVIĆ, Editor, *Sensitivity Methods in Control Theory*, Pergamon Press, New York, 1966, xiii + 442 pp., 24 cm. Price $13.50.

This volume contains 30 papers that were presented at the International Symposium on Sensitivity Analysis, Dubrovnik, Yugoslavia in 1964 under the auspices of the Theory Committee of IFAC and is divided into five sections: I. Basic approaches, II. Sensitivity functions, III. Compensation of parameter variations, IV. Synthesis of insensitive structures, and V. Sensitivity and optimality. Among the basic approaches proposed are stability theory (I. Gumowski and Ya. Z. Tsyphin), invariant imbedding (R. Bellman, R. Kalaba, R. Sridhar), optimality and game theory (P. Dorato, R. F. Drenick), sensitivity operators for linear problems (W. R. Perkins, J. B. Cruz, Jr.), and computer methods and simulation (R. Tomović). Section II is concerned with quantitative measures of sensitivity and their use in the design of systems. Here one sees special methods for special problems. One paper (J. Vidal, W. J. Karplus, and G. Kaludjian) discusses the correction of quantization errors in hybrid computer systems. The systems in Section III are either "self-adjusting" (also called "adaptive") or "insensitive" to parameter variations and various design schemes are proposed. One of these schemes is the use of what the Russians call "the theory of invariance." "Invariance" equals complete insensitivity to a variation of certain parameters. Section IV deals exclusively with the design of insensitive systems. The sensitivity of optimal control systems is discussed in Section V.

As admitted by the editor in the Preface and substantiated by the papers, there is "no unified opinion" as to the meaning of "sensitivity" even though, like the word "stability," everyone has a feeling for what it means. The view of Bellman is that sensitivity is a concept which cannot be defined except relative to a system and what is expected of it. And this suggests mentioning that an important problem is not when is optimality insensitive (Section V) but when does optimality imply insensitivity in this pragmatic sense.

The Proceedings may disappoint someone looking for applicable results but it should be remembered that sensitivity theory is neither well defined nor well developed. The volume contributes to an understanding of the state of development of the theory, its objectives, and proposed methods of attack.

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63[K, X].—J. KEILSON, *Green's Function Methods in Probability Theory*, Hafner Publishing Co., New York, 1965, viii + 220 pp., 22 cm. Price $6.50.

The central subject of the book is the theory of one-dimensional spatially and temporally homogeneous Markov processes, both unrestricted and in the presence of absorbing barriers. The theory is comparable to that in J. H. B. Kemperman's *The Passage Problem for a Stationary Markov Chain*, University of Chicago Press, Chicago, Ill., 1961. Just as Kemperman, the author relies heavily on the analysis of characteristic and moment generating functions in the complex domain. This leads to unified complex variable proofs of central limit theorems, renewal theorems, and other asymptotic results required in the applications to queues, dams, risk, and inventory problems. The emphasis on Green's functions is partly a matter of terminology, as the author so labels all transition measures (which indeed are Green's functions of the space-time process). The book is more highly recommended to the reader engaged in sophisticated applications than to the serious beginner in stochastic processes.

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64[L].—HENRY E. FETTIS & JAMES C. CASLIN, *An Extended Table of Zeros of Cross Products of Bessel Functions*, Report ARL 66-0023, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1966, v + 126 pp., 28 cm. [Copies obtainable from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.]

This useful report presents 10D tables of the first five roots of the equations: (a) $J_0(\alpha)Y_0(k\alpha) - Y_0(\alpha)J_0(k\alpha) = 0$, (b) $J_1(\alpha)Y_1(k\alpha) - Y_1(\alpha)J_1(k\alpha) = 0$, (c) $J_0(\alpha)Y_1(k\alpha) - Y_0(\alpha)J_1(k\alpha) = 0$.

In particular, Table 1a gives such roots of Eq. (a) for $k = 0.01(0.01)0.99$, while Table 1b gives the corresponding normalized roots $\gamma_n = (1 - k)\alpha_n/(n\pi)$, which are better adapted to interpolation, as originally observed by Bogert [1].

The same information for Eq. (b) is given in Tables 2a and 2b. In Tables 3a and 4a we find the corresponding roots of Eq. (c) for the respective ranges $k = 0.01(0.01)0.99$ and $k = 1.01(0.01)20$; the corresponding normalized roots $\gamma_n = |1 - k|\alpha_n/[(n - \frac{1}{2})\pi]$ appear in Tables 3b and 4b. The last two tables (5a and 5b) give the roots of Eq. (c) and their normalized equivalents for $k^{-1} = 0.001(0.001)0.050$.

As the authors note, because of symmetry it suffices for Eq. (a) and Eq. (b) to tabulate the roots corresponding to $0 < k < 1$.

The values of the roots γ_n were calculated by the method of false position on an IBM 7094 system, subject to the requirement that the corresponding values of the left member of the appropriate equation not exceed 10^{-16} numerically. These values of γ_n were then converted to the corresponding values of α_n, and both sets of data were then rounded to 10D.

Previously published tables of this kind have been very limited in scope and precision; one of the most extensive of these appears in a compilation (to 5D and 8D) on page 415 of the NBS *Handbook* [2]. The present authors have announced [3] a number of errors therein as a result of their more extensive calculations.

This reviewer has compared entries in Table 2a with the corresponding 5D approximations appearing in the table of roots of $\Delta_0(\xi) = 0$ in a recent paper by Bauer [4]. The accuracy of at least 4D claimed by Bauer is now confirmed.

In a brief introduction the authors show how such equations involving Bessel functions arise in certain boundary-value problems. This is elaborated upon in Appendix 1, which shows the relation of the tables to the solution of a problem in heat conduction involving three sets of boundary conditions.

An asymptotic series for the higher roots of the equation $J_p(\alpha)Y_q(k\alpha) -$